$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \qquad \text{(Friedmann eq.)}$$

Matter-domination : $\rho \propto \frac{1}{a^3}$

If the universe is flat $(k=0)$

$$\Rightarrow \frac{\dot{a}^2}{a^2} \propto \rho \propto \frac{1}{a^3}$$

$$\Rightarrow \dot{a} \propto a^{-1/2} \quad \Rightarrow a^{1/2}\dot{a} \propto \text{const.}$$

$$\Rightarrow \frac{2}{3} a^{3/2} \propto (\text{const.})_1 t + (\text{const.})_2$$

Then, we say (define) $a = 0$ at $t = 0$

$$\Rightarrow a^{3/2} \propto t$$

$$\Rightarrow a = b\, t^{2/3}$$

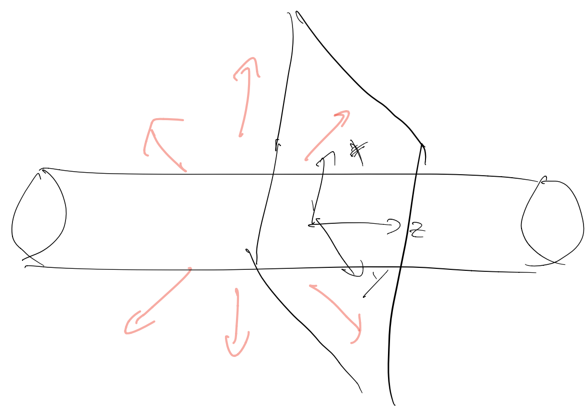
PSet 3, Problem 1 e)

In part d), you should have obtained

$$\ddot{a} = -2\pi G \rho a \qquad / \; \dot{a}$$

$\rho = \frac{\rho_i}{a^2}$

$$\Rightarrow \dot{a}\left\{\ddot{a} + \frac{2\pi G \rho_i}{a}\right\} = 0$$



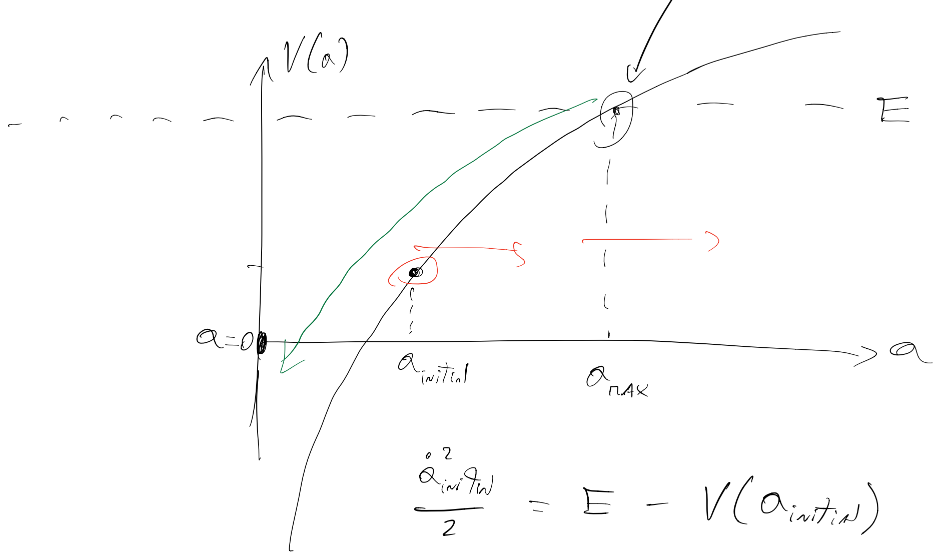
$$\frac{d}{dt}\left\{\frac{1}{2}\dot{a}^2 + 2\pi G \rho_i \ln a\right\} = \frac{dE}{dt} = 0$$

$$E = \frac{\dot{a}^2}{2} + 2\pi G \rho_i \ln(a)$$

// constant.

$$V(a) = 2\pi G \rho_i \ln(a)$$

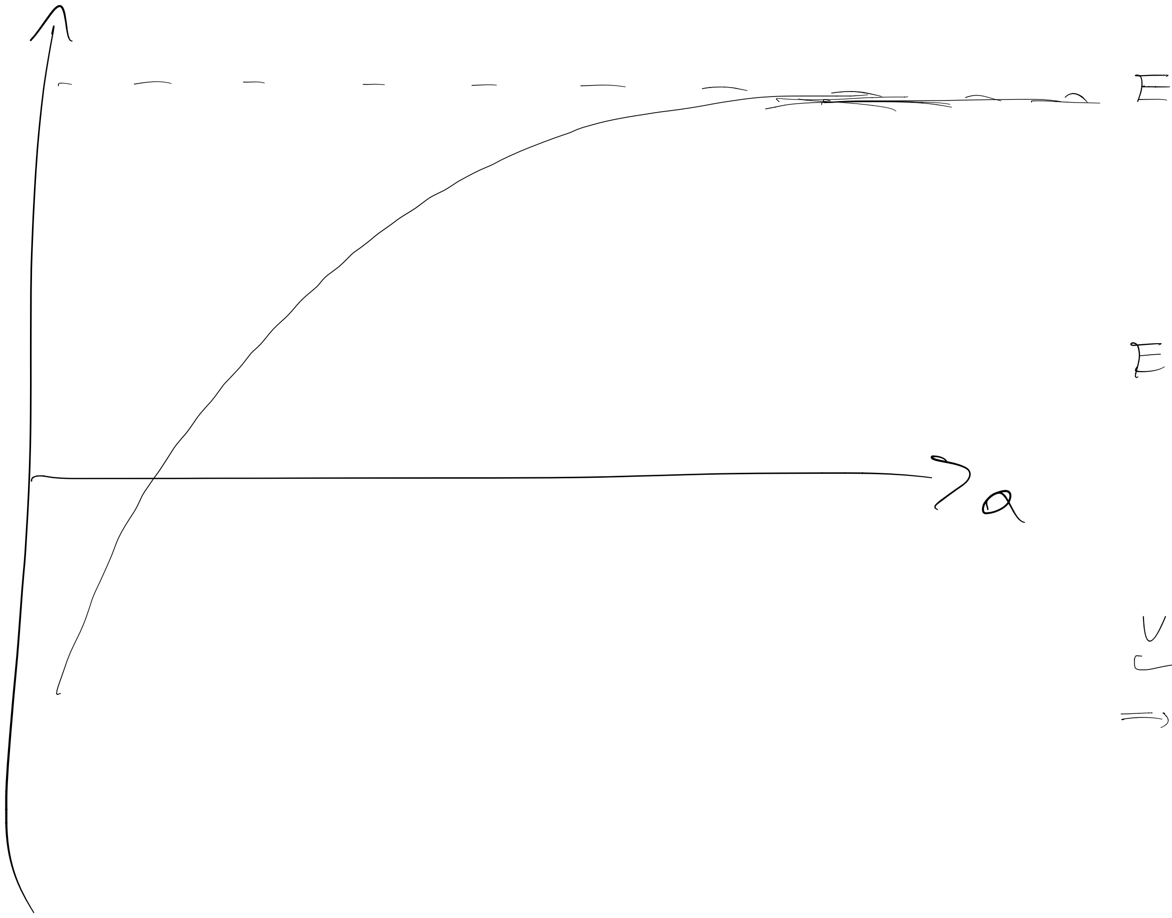
$$\dot{a} = 0$$



$$\frac{\dot{a}_{initial}^2}{2} = E - V(a_{initial})$$

If our universe grows forever, then $a(t)$ must reach arbitrarily large values

$$\ddot{a} = -2\pi G \rho a \implies \text{The universe is always decelerating}$$



$$E = \frac{\dot{a}^2}{2} + V(a)$$

$$\underbrace{V(a) \propto -a^{-1}}$$

$$\implies V \xrightarrow[a \to \infty]{} 0$$